

Two-photon exchange in dispersion approach

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We calculate two-photon exchange amplitude for the elastic electron-proton scattering in the framework of dispersion relations. The imaginary part of the amplitude is determined by unitarity. Since in the unitarity relation intermediate states are on shell, off-shell form factors are not needed for the calculation. The real part is then evaluated using analytical properties of the amplitude. The expression for the elastic contribution to the amplitude, obtained in our approach, differs from the results of traditional calculations with on-shell form factors. Nevertheless, numerically the difference is minor for Q^2 up to 6 GeV^2 .

I. INTRODUCTION

The precision level of present-day electron-proton scattering experiments makes it necessary to take into account effects beyond Born approximation, such as two-photon exchange (TPE). TPE can be seen in various observables in wide kinematical range; in particular it influences proton radius measurements [1], generates non-zero transverse beam spin asymmetry [2], and, the most important, TPE corrections play crucial role in reconciliation of different measurements of proton form factors (FFs) at high Q^2 [3]. Clearly, such corrections are also required for analysis of data from upcoming measurements at higher Q^2 [4].

The TPE diagram (Fig. 1) for elastic ep scattering differs from similar diagram in QED in two ways. First, the proton is not a point-like object, thus there are some non-trivial FFs at γp vertices. Second, the interaction of the proton with virtual photon may lead to excitation of inelastic intermediate states, such as πp , Δ resonance, and so on.

At present, the calculations exist for elastic intermediate state [5, 6], and for a number of resonances [7]¹. Though the evaluation of loop integral in these papers was almost perfect, the weak point of all such calculations is the starting expression for the TPE diagram (here we consider the elastic contribution, but the contribution of resonances may be studied similarly). This expression results from the contraction of “leptonic” and “hadronic” parts

$$i\mathcal{M}^{(\text{naive})} = \int \frac{(4\pi\alpha)^2}{q_1^2 q_2^2} L^{\mu\nu} H_{\mu\nu} \frac{d^4 k''}{(2\pi)^4} \quad (1)$$

where $q_1 = k - k''$, $q_2 = k'' - k'$, the leptonic part $L_{\mu\nu}$ comes from QED

$$L_{\mu\nu} = \bar{u}' \gamma_\mu \frac{\hat{k}'' + m}{k''^2 - m^2} \gamma_\nu u \quad (2)$$

but the hadronic part $H_{\mu\nu}$ is only *guessed* to be

$$H_{\mu\nu} = \bar{U}' \Gamma_\mu(q_2) \frac{\hat{p}'' + M}{p''^2 - M^2} \Gamma_\nu(q_1) U + \bar{U}' \Gamma_\nu(q_1) \frac{\hat{p}'' + M}{\tilde{p}''^2 - M^2} \Gamma_\mu(q_2) U \quad (3)$$

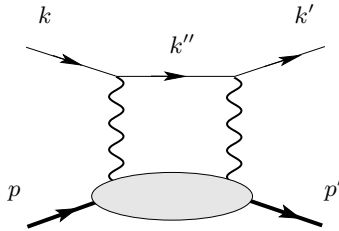


FIG. 1: TPE diagram

¹ There are also partonic model calculations, appropriate at large Q^2 and energy [8]. This approach will not be considered further.

where $p'' = p + q_1$, $\tilde{p}'' = p + q_2$ and $\Gamma_\mu(q)$ is the amplitude of proton interaction with electromagnetic field, written in the form

$$\Gamma_\mu(q) = \gamma_\mu F_1(q^2) - \frac{1}{4M} F_2(q^2) [\gamma_\mu, \hat{q}] \quad (4)$$

The above-described form of the TPE amplitude was used by many authors from Bodwin and Yennie in 1988 [9] to the latest papers [5, 6]. The justification for such choice of $H_{\mu\nu}$ is the following: first, it gives the expected result if the intermediate proton is on-shell, or more precisely, it has correct residues at $p''^2 = M^2$ and $\tilde{p}''^2 = M^2$, and second, this expression is gauge-invariant, i.e.

$$q_{1\nu} H_{\mu\nu} = q_{2\mu} H_{\mu\nu} = 0 \quad (5)$$

However, (3) is not the only expression with such properties. One can, for instance, add to F_2 an arbitrary function which vanishes at $p''^2 = M^2$, like

$$F_2(q^2) \rightarrow F_2(q^2) + (p''^2 - M^2) f(q^2) \quad (6)$$

We should emphasize that, if we are dealing with the elastic contribution only, than the choice of $H_{\mu\nu}$ is somewhat a matter of convention, since any change of the elastic contribution may be compensated by appropriate redefinition of the inelastic one. Nevertheless, it is desirable to have clear, unambiguous, and easy-for-calculation definition for contribution of each intermediate state.

The modification of γp vertex at $p''^2 \neq M^2$, which of course can be more general than the example shown above, is usually referred to as introduction of proton off-shell FFs. The uncertainty of these FFs is believed to be the main source of theoretical uncertainty in TPE amplitudes [3]. On the other hand, such FFs are not directly measurable, just because off-shell proton cannot be a final state. Hence to take into account off-shell behaviour, one cannot rely on experimental data but instead must use some nucleon model. This is undesirable, since the result will be model-dependent.

In the current paper we propose a consistent approach to calculation of TPE, in which the use of “off-shell” FFs is avoided. The approach is based on the dispersion relations. At first, the absorptive part of the amplitude is calculated using unitarity. Thus only “on-shell” FFs are needed to evaluate it. Then the whole amplitude is reconstructed by dispersion relations. Since this operation is linear, contributions from different intermediate states may be treated separately.

II. THE AMPLITUDES

We follow the notation of Refs.[5, 10, 11]. In particular, we define $P = (p + p')/2$, $K = (k + k')/2$ and $t = q^2$, $\nu = s - u = 4PK$, where s , t and u are Mandelstam variables. The electron and proton masses are m and M , respectively. In the present section (but not in the whole paper) the electron mass is neglected.

The general-case elastic ep scattering amplitudes is conveniently written as [12]

$$\mathcal{M} = \frac{4\pi\alpha}{q^2} \bar{u}' \gamma_\mu u \cdot \bar{U}' \left(\gamma^\mu \tilde{F}_1 - \frac{1}{4M} [\gamma^\mu, \hat{q}] \tilde{F}_2 + \frac{P^\mu}{M^2} \hat{K} \tilde{F}_3 \right) U \quad (7)$$

In Ref. [10] the following set of amplitudes was introduced

$$\begin{aligned} \mathcal{G}_E &= \tilde{F}_1 - \tau \tilde{F}_2 + \nu \tilde{F}_3 / 4M^2 \\ \mathcal{G}_M &= \tilde{F}_1 + \tilde{F}_2 + \varepsilon \nu \tilde{F}_3 / 4M^2 \\ \mathcal{G}_3 &= \nu \tilde{F}_3 / 4M^2 \end{aligned} \quad (8)$$

which “diagonalizes” the cross-section

$$d\sigma = \frac{2\pi\alpha^2 dt}{E^2 t} \frac{1}{1 - \varepsilon} \left(\varepsilon |\mathcal{G}_E|^2 + \tau |\mathcal{G}_M|^2 + \tau \varepsilon^2 \frac{1 - \varepsilon}{1 + \varepsilon} |\mathcal{G}_3|^2 \right) \quad (9)$$

In the above equations, E is initial electron lab. energy, $\tau = -t/4M^2$ and $\varepsilon = [\nu^2 + t(4M^2 - t)]/[\nu^2 - t(4M^2 - t)]$. Since the amplitude \mathcal{G}_3 vanishes in Born approximation and hence is $O(\alpha)$, the last term in (9) is negligibly small and we have

$$d\sigma \approx d\sigma_0 (\varepsilon |\mathcal{G}_E|^2 + \tau |\mathcal{G}_M|^2) \quad (10)$$

similarly to Rosenbluth formula, except that \mathcal{G}_E and \mathcal{G}_M are ε -dependent.

However to make use of the dispersion relations, we need amplitudes, free from kinematical u and s singularities and zeros. Such amplitudes are easily constructed by consideration of annihilation channel. The helicity amplitudes of the process $e^-e^+ \rightarrow p\bar{p}$ are

$$T_{++} = 4\pi\alpha \cdot 2i \cos^2 \theta/2 \left(\sqrt{\tau(1+\tau)}\tilde{F}_3 + \tilde{F}_m + \nu\tilde{F}_3/4M^2 \right) \quad (11)$$

$$T_{--} = 4\pi\alpha \cdot 2i \sin^2 \theta/2 \left(\sqrt{\tau(1+\tau)}\tilde{F}_3 - \tilde{F}_m - \nu\tilde{F}_3/4M^2 \right) \quad (12)$$

$$T_{+-} = T_{-+} = 4\pi\alpha \cdot \frac{2M}{\sqrt{t}} \sin \theta \left(\tilde{F}_e + \nu\tilde{F}_3/4M^2 \right) \quad (13)$$

where $\tilde{F}_e = \tilde{F}_1 - \tau\tilde{F}_2$, $\tilde{F}_m = \tilde{F}_1 + \tilde{F}_2$, and θ is t -channel scattering angle,

$$\cos \theta = -\nu/\sqrt{-t(4M^2 - t)} \quad (14)$$

The subscripts of the quantity $T_{\lambda\bar{\lambda}}$ indicate the signs of proton and antiproton helicities, respectively, while the electron and positron helicities are $+1/2$ and $-1/2$. Computing the scattering channel cross-section

$$\frac{d\sigma}{dt} = \frac{1}{64\pi M^2 E^2} \cdot \frac{1}{2} (|T_{++}|^2 + |T_{--}|^2 + 2|T_{+-}|^2) \quad (15)$$

we return to the formula (9). Each of the $T_{\lambda\bar{\lambda}}$ contains a kinematical factor of $\sin^{|\lambda+\bar{\lambda}-1|} \frac{\theta}{2} \cos^{|\lambda+\bar{\lambda}+1|} \frac{\theta}{2}$ (see e.g. Ref. [13]). The amplitudes free from kinematical singularities are obtained after removing these factors, i.e.

$$\sqrt{\tau(1+\tau)}\tilde{F}_3 \pm \left(\tilde{F}_m + \nu\tilde{F}_3/4M^2 \right) \quad \text{and} \quad \tilde{F}_e + \nu\tilde{F}_3/4M^2$$

or equivalently

$$G_1 \equiv \mathcal{G}_E = \tilde{F}_e + \nu\tilde{F}_3/4M^2, \quad G_2 = \tilde{F}_m + \nu\tilde{F}_3/4M^2, \quad G_3 \equiv \tilde{F}_3 \quad (16)$$

The amplitudes G_n satisfy fixed- t dispersion relations

$$\pi G_n(\nu) = \int_{\nu_{th}}^{\infty} \frac{\text{Im } G_n(\nu' + i0)}{\nu' - \nu} d\nu' - \int_{-\infty}^{-\nu_{th}} \frac{\text{Im } G_n(\nu' - i0)}{\nu' - \nu} d\nu' \quad (17)$$

and consequently, vanish at $\nu \rightarrow \infty$. Under crossing $\nu \rightarrow -\nu$ two first amplitudes are odd and the last is even:

$$G_{1,2}(-\nu) = -G_{1,2}(\nu), \quad G_3(-\nu) = G_3(\nu). \quad (18)$$

III. CALCULATION PROCEDURE

A. Imaginary part

The imaginary part of the scattering amplitude can be calculated via unitarity condition

$$T_{if}^* - T_{fi} = \sum_n T_{fn} T_{in}^* \quad (19)$$

or graphically

$$2 \text{Im} \left(\text{Diagram 1} \right) = \int \frac{d^3 \vec{k}''}{2k_0''} \sum_h \left(\text{Diagram 2} \right) \times \left(\text{Diagram 3} \right) \quad (20)$$

where we have replaced T -matrix elements in the r.h.s. by their Born (one-photon exchange) approximations. Thus obtained is exactly the absorptive part of the TPE amplitude.

Eq.(20) allows for natural and unambiguous classification of different contributions to $\text{Im } G_n$, according to intermediate hadronic states h . The term with $h = \text{proton}$ will be called elastic contribution, the term with $h = \Delta(1232)$ will be the Δ resonance contribution and so on. Since the intermediate states appearing in the unitarity condition are real (“on-shell”) particles, it is sufficient to know on-shell transition amplitudes of these states to calculate $\text{Im } G_n$. Thus in particular the knowledge of proton “off-shell” FFs is not needed.

The reconstruction of the $\text{Re } G_n$ from $\text{Im } G_n$ by dispersion integral is linear operation, therefore we may introduce a natural definition of elastic contribution to the whole amplitude as the quantity yielded by dispersion relation applied to the elastic part of $\text{Im } G_n$, and similarly for other contributions.

B. Reconstruction of the real part

From now on we consider the elastic contribution only. Such contribution to the imaginary part of invariant amplitudes G_n can be written in the form

$$\text{Im } G_n^{(\text{el})} = -\frac{\alpha}{2\pi} \sum_{i,j=1}^2 \int \bar{F}_i(t_1) \bar{F}_j(t_2) A_{n,ij}(\nu, t_1, t_2) \theta(k_0'') \delta(k''^2 - m^2) \theta(p_0'') \delta(p''^2 - M^2) d^4 k'' \quad (21)$$

where $\bar{F}_i(t) = F_i(t)/(t - \lambda^2)$, $A_{n,ij}$ is a polynomial in t_1, t_2 and a rational function of ν (it may have poles in ν ; the explicit expression for $A_{n,ij}$ is given in Appendix A). The θ - and δ - functions ensure that intermediate particles are on-shell. The straightforward way of further calculation is to insert $\text{Im } G_n^{(\text{el})}$ into the dispersion integral (17) and evaluate it. However this is not an easy task. For example, Eq.(21) for imaginary part is valid only in physical region $\nu \geq \sqrt{-t(4M^2 - t)}$, but the dispersion integral involves all ν values above the threshold $\nu = \nu_{th}$ (corresponding to $s = (M+m)^2$), thus before it can be evaluated we must first find an analytical continuation of (21) into the unphysical region. Though such analytical continuation is unique, it is hard to write it down in a compact form. So we will use an easier roundabout way.

The amplitude G_n is an analytical function of ν with two branch cut discontinuities along the real axis: from $-\infty$ to $-\nu_{th}$ and from ν_{th} to $+\infty$. As implied by Eq.(17), it can be written as a sum of two parts, direct and crossed box amplitudes

$$G_n(\nu) = G_{n,\text{box}}(\nu) + G_{n,\text{xbox}}(\nu) \quad (22)$$

with each of them having only one discontinuity, box from ν_{th} to $+\infty$ and crossed box from $-\infty$ to $-\nu_{th}$. Direct and crossed box amplitudes are related by

$$G_{n,\text{box}} = \pm G_{n,\text{xbox}}(-\nu) \quad (23)$$

where \pm is chosen according to (18). Thus to reconstruct G_n it is sufficient to find $G_{n,\text{box}}$.

To do this, we note that if we find any function with the following properties:

- 1) it has no singularities except the branching point at $s = (M+m)^2$,
- 2) its branch cut discontinuity is $2i \text{Im } G_n^{(\text{el})}$, with $\text{Im } G_n^{(\text{el})}$ given by Eq.(21),
- 3) it vanishes as $s \rightarrow \infty$,

then such function necessarily coincides with the sought amplitude (otherwise their difference would be non-trivial bounded whole function, which is impossible).

The analytical structure of FFs is such that

$$\bar{F}_i(t) = \frac{1}{\pi} \int_{\lambda^2}^{\infty} \frac{\text{Im } \bar{F}_i(t')}{t' - t} dt' \quad (24)$$

in other words, the FFs in the Eq.(21) are some linear combinations of a single poles $\frac{1}{t-a}$ with $a > 0$. Using the decomposition (24), we may obtain

$$\sum_{i,j=1}^2 \bar{F}_i(t_1) \bar{F}_j(t_2) A_{ij}(\nu, t_1, t_2) = \int_{\lambda^2}^{\infty} da \int_{\lambda^2}^{\infty} db \frac{c(\nu, a, b)}{(t_1 - a)(t_2 - b)} \quad (25)$$

and rewrite Eq.(21) as

$$\text{Im } G_n^{(\text{el})} = \int_{\lambda^2}^{\infty} da \int_{\lambda^2}^{\infty} db c(\nu, a, b) \int \frac{1}{(t_1 - a)(t_2 - b)} \theta(k_0'') \delta(k''^2 - m^2) \theta(p_0'') \delta(p''^2 - M^2) d^4 k'' \quad (26)$$

Consider the function

$$I_4(s, t; a, b) = \int \frac{i d^4 k''}{(t_1 - a)(t_2 - b)(k''^2 - m^2)(p''^2 - M^2)} \quad (27)$$

It is well-known that this is an analytic function of s everywhere except the branch cut from $s = (M + m)^2$ to $+\infty$. Its discontinuity across the cut is

$$\Delta I_4 = 2i \text{Im } I_4 = \int \frac{-4i\pi^2}{(t_1 - a)(t_2 - b)} \theta(k_0'') \delta(k''^2 - m^2) \theta(p_0'') \delta(p''^2 - M^2) d^4 k'' \quad (28)$$

which is exactly the innermost integral in (26). Thus if the coefficients $c(\nu, a, b)$ were independent of ν , the whole TPE amplitude would be obtained by substitution

$$\theta(k_0'') \delta(k''^2 - m^2) \theta(p_0'') \delta(p''^2 - M^2) \rightarrow \frac{1}{2i\pi^2} \frac{1}{(k''^2 - m^2)(p''^2 - M^2)} \quad (29)$$

under the integral, yielding

$$\tilde{G}_n^{(\text{el})}(\nu) = \frac{i\alpha}{4\pi^3} \sum_{i,j=1}^2 \int \bar{F}_i(t_1) \bar{F}_j(t_2) A_{n,ij}(\nu, t_1, t_2) \frac{d^4 k''}{(k''^2 - m^2)(p''^2 - M^2)} \quad (30)$$

But actually quantities $A_{n,ij}(\nu, t_1, t_2)$ and thus $c(\nu, t_1, t_2)$ have poles at the boundary of the physical region $\nu = \pm\nu_0 = \pm\sqrt{-t(4M^2 - t)}$ (see explicit expressions in Appendix A). Because of this function $\tilde{G}_n^{(\text{el})}$, constructed by Eq.(30), satisfy conditions 2) and 3) but don't satisfy 1) since it has unphysical poles at $\nu = \pm\nu_0$.

To remove these poles we may simply subtract the principal part of \tilde{G}_n Laurent series expansion about $\nu = \pm\nu_0$

$$G_{n,\text{box}}^{(\text{el})}(\nu) = \tilde{G}_n^{(\text{el})}(\nu) - \sum_{r=0}^{N-1} \frac{1}{r!} \frac{g_{r+}}{(\nu - \nu_0)^{N-r}} - \sum_{r=0}^{N-1} \frac{1}{r!} \frac{g_{r-}}{(\nu + \nu_0)^{N-r}} \quad (31)$$

where N is degree of the pole (actually 1 or 2) and

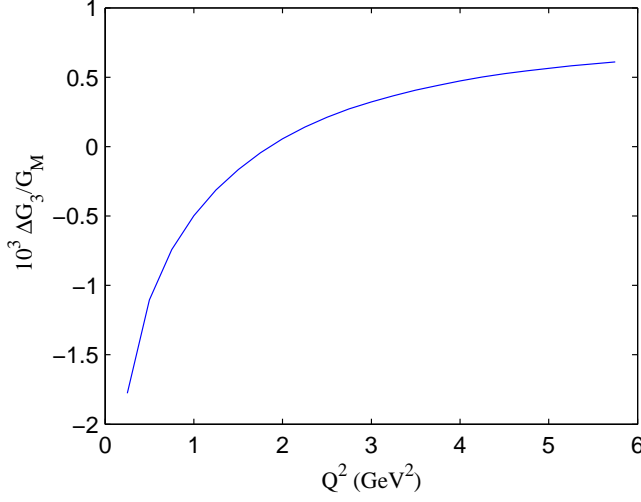
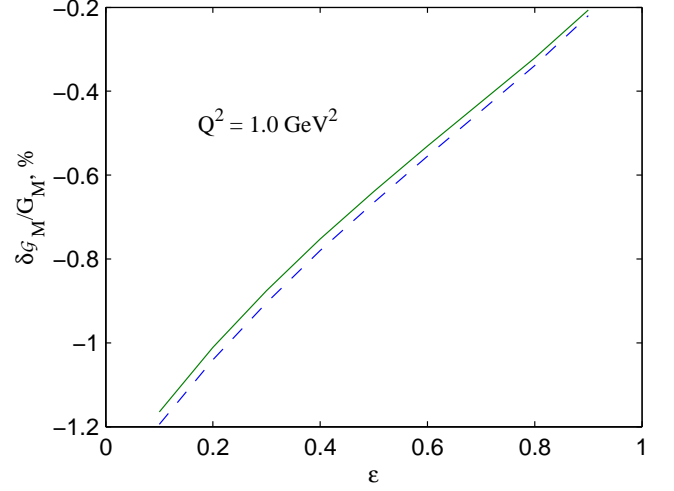
$$g_{r\pm} = \left. \frac{\partial^r}{\partial \nu^r} (\nu \mp \nu_0)^N \tilde{G}_n^{(\text{el})}(\nu) \right|_{\nu=\pm\nu_0} \quad (32)$$

Since the subtracted function is meromorphic (has no branching points) and vanish at $\nu = \infty$, the properties 2) and 3) hold true and in addition, the obtained function $G_{n,\text{box}}^{(\text{el})}(\nu)$ is regular at $\nu = \pm\nu_0$. So the requirement 1) is also satisfied. Therefore $G_{n,\text{box}}^{(\text{el})}(\nu)$ is the sought amplitude.

In summary, the evaluation of the TPE amplitude proceeds as follows:

- 1) construct the expression for the imaginary part in the form (21).
- 2) obtain the quantity \tilde{G}_n , Eq.(30), by substitution according to Eq.(29).
- 3) subtract unphysical poles at $\nu = \pm\nu_0$, Eq.(31).
- 4) perform (anti)symmetrization with respect to ν , i.e. add crossed box amplitude.

Due to decomposition (25) the quantity \tilde{G}_n can be written as a linear combination of functions $I_4(s, t; a, b)$ with different a and b . This is especially useful if FFs are parameterized as a discrete sum of a single poles (such an approach was used in Ref. [6]). To perform the subtraction of unphysical poles one needs to know the value of function I_4 and its derivative at $\nu = \pm\nu_0$. They can be expressed via integrals similar to (27) with $k'' - m^2$ or $p''^2 - M^2$ or both dropped (such integrals were denoted I_1, I_2, I_3 in Ref. [5]). Some useful relations between them are given in Appendix B. With these relations, one may compare the expression for elastic part of TPE amplitude, obtained in the dispersion approach, with the “naive” result (1-3).

FIG. 2: The amplitude change ΔG_3 .FIG. 3: The TPE amplitude $\delta\mathcal{G}_M$ obtained in old (dashed line) and new (solid line) approach.

IV. RESULTS AND CONCLUSIONS

After performing the above-described procedure, we have obtained the following results for the elastic contributions to the invariant amplitudes G_n . The expressions for G_1 and G_2 remain the same as in the “naive” approach, Eqs.(1-3)

$$G_1 = G_1^{(\text{naive})}, \quad G_2 = G_2^{(\text{naive})} \quad (33)$$

The expression for G_3 is different:

$$G_3 = G_3^{(\text{naive})} + \Delta G_3(t) \quad (34)$$

where

$$\Delta G_3(t) = \frac{i\alpha}{4\pi^3 t} \int \frac{F_2(t_1)F_2(t_2)}{t_1 t_2} \left(t_1 + t_2 + 3t - \frac{2t_1 t_2}{k'^2 - m^2} \right) d^4 k'' \quad (35)$$

The whole scattering amplitude may be written as

$$\mathcal{M} = \mathcal{M}^{(\text{naive})} + \frac{4\pi\alpha}{q^2 M^2} \bar{u}' \gamma^\mu u \bar{U}' (P_\mu \hat{K} - PK\gamma_\mu) U \cdot \Delta G_3(t) \quad (36)$$

Since the quantities that contribute to the cross-section up to the order $O(\alpha)$ are

$$\mathcal{G}_E = G_1 \quad \text{and} \quad \mathcal{G}_M = G_2 - \frac{\nu}{4M^2} (1 - \varepsilon) G_3 \quad (37)$$

(see Eq.(10)), with new expression (34) for G_3 TPE corrections to the cross-section will differ from those in “naive” approach, since

$$\mathcal{G}_M = \mathcal{G}_M^{(\text{naive})} - \sqrt{\tau(1+\tau)} \sqrt{1-\varepsilon^2} \Delta G_3(t) \quad (38)$$

Moreover, the affected amplitude, \mathcal{G}_M , is exactly the quantity which is responsible for the discrepancy between Rosenbluth and polarization transfer methods in the measurements of proton FFs [11].

The numerical calculation, however, shows that the addition to \mathcal{G}_M is very small (Figs. 2 and 3). Therefore most of the results obtained starting from “naive” expression for the amplitude will remain unchanged. In particular, we checked what the low- Q^2 behaviour is the same as reported in Ref.[10], since the addition to \mathcal{G}_M vanishes at $Q^2 \rightarrow 0$. Nevertheless, the proton off-shell form factors problem is overcome: they are not needed to calculate TPE amplitudes in our approach.

APPENDIX A

The coefficients $A_{n,ij}$ may be computed in the following way. First, we write down the standard expression for the absorptive part of the amplitude (elastic contribution)

$$\begin{aligned} \text{Im } \mathcal{M} = & \frac{1}{8\pi^2} \int \frac{(4\pi\alpha)^2}{q_1^2 q_2^2} \bar{u}' \gamma_\mu (\hat{k}'' + m) \gamma_\nu u \cdot \bar{U}' \Gamma_\mu(q_2)(\hat{p}'' + M) \Gamma_\nu(q_1) U \times \\ & \times \theta(k_0'') \delta(k''^2 - m^2) \theta(p_0'') \delta(p''^2 - M^2) d^4 k'' \end{aligned} \quad (\text{A1})$$

Then we decompose it into scalar invariant amplitudes according to Eq.(7) (the equation for $\text{Im } \mathcal{M}$ contains $\text{Im } \tilde{F}_n$ instead of \tilde{F}_n in the r.h.s.). Since $\text{Im } \tilde{F}_n$ are scalars, they will depend on scalar combinations of p, k, p', k', p'' and k'' . But due to "onshellness" of the intermediate particles we have $p''^2 = M^2$ and $k''^2 = m^2$, and other scalar products can be expressed via q_1^2, q_2^2, ν and t . The vertex functions Γ_μ and Γ_ν contain FFs, so the resulting expression will be quadratic in FFs. The $\text{Im } G_n$ are obtained as linear combinations of $\text{Im } \tilde{F}_n$, Eq.(16).

Below $A_{n,ij}$ are written in a matrix notation, $A_n = \begin{pmatrix} A_{n,11} & A_{n,12} \\ A_{n,21} & A_{n,22} \end{pmatrix}$. In these formulas $t_p = t_1 + t_2 - t$, $t_m = t_1 - t_2$ and $\nu_0^2 = -t(4M^2 - t)$.

$$\begin{aligned} A_1 = & t(\nu - t) \left\{ \frac{1}{2} + t_p \frac{4M^2 + 2\nu - t + t_p}{4(\nu^2 - \nu_0^2)} \right\} \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} + \frac{tt_m(\nu - t)}{16M^2} \left\{ 2 + t_p \frac{4M^2 + \nu - t}{\nu^2 - \nu_0^2} \right\} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + \\ & + \frac{t(\nu - t)}{16M^2} \left\{ 2t + t_p + t_p \frac{(2t + t_p)(4M^2 + \nu - t) + 2M^2 t_p}{\nu^2 - \nu_0^2} \right\} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \\ & + \left\{ \frac{tt_p(\nu - t)(2M^2 t_p - \nu(\nu - t))}{16M^2(\nu^2 - \nu_0^2)} - \frac{tt_1 t_2}{4M^2} \right\} \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} \end{aligned} \quad (\text{A2})$$

$$\begin{aligned} A_2 = & (\nu - t) \left\{ \frac{t}{2} + t_p \frac{t(4M^2 + \nu - t) - t_p(2M^2 - t)}{2(\nu^2 - \nu_0^2)} + tt_p^2 \frac{(4M^2 - t)(2M^2 + \nu - t)}{(\nu^2 - \nu_0^2)^2} \right\} \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} - \\ & - \frac{t(\nu - t)t_p}{16M^2} \left\{ 1 + \frac{(4M^2 - t + t_p)(\nu - t) + 2M^2 t_p}{\nu^2 - \nu_0^2} + 2(4M^2 - t)(\nu - t)t_p \frac{2M^2 + \nu - t}{(\nu^2 - \nu_0^2)^2} \right\} \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} + \\ & + \left\{ t_p(t + t_p) \frac{\nu(\nu - t)}{4(\nu^2 - \nu_0^2)} - t_1 t_2 \right\} \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} + (\nu - t)t_m \left\{ \frac{1}{2} + \frac{\nu t_p}{4(\nu^2 - \nu_0^2)} \right\} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + \\ & + \frac{(4M^2 - t)(\nu - t)}{\nu^2 - \nu_0^2} t_1 t_2 \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} + \frac{t(4M^2 - t)(2M^2 + \nu - t)}{4M^2(\nu^2 - \nu_0^2)} t_1 t_2 \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \end{aligned} \quad (\text{A3})$$

$$\begin{aligned} A_3 = & (\nu - t)t_p \left\{ \frac{t_p(6M^2 + \nu - 3t) - t(\nu - t)}{4(\nu^2 - \nu_0^2)} - tt_p \frac{(3M^2 - t)\nu + (4M^2 - t)(M^2 - t)}{(\nu^2 - \nu_0^2)^2} \right\} \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} - \\ & - \nu t_1 t_2 \frac{2M^2 + \nu - t}{\nu^2 - \nu_0^2} \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} + t_p(t + t_p) \frac{M^2(\nu - t)}{\nu^2 - \nu_0^2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \frac{M^2(\nu - t)t_m t_p}{\nu^2 - \nu_0^2} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + \\ & + \left\{ 4M^2 t_1 t_2 \frac{4M^2 + \nu - t}{\nu^2 - \nu_0^2} - 2M^2(\nu - t)^2 t_p \frac{(4M^2 + \nu - t)t_p + \nu^2 - \nu_0^2}{(\nu^2 - \nu_0^2)^2} \right\} \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} \end{aligned} \quad (\text{A4})$$

APPENDIX B

It is convenient to use Breit frame, in which

$$q = (0, 0, 0, \sqrt{-t}), \quad P = \left(\frac{1}{2} \sqrt{4M^2 - t}, 0, 0, 0 \right), \quad K = \frac{1}{2\sqrt{4M^2 - t}} \left(\nu, \sqrt{\nu^2 - \nu_0^2}, 0, 0 \right) \quad (\text{B1})$$

At $\nu = \nu_0$ all three vectors have only time- and z - components. The components p_0'' and p_z'' of the vector p'' can be expressed via t_1, t_2 and p''^2 as

$$p_z'' = \frac{t_2 - t_1}{2\sqrt{-t}}, \quad p_0'' = \frac{1}{2\sqrt{4M^2 - t}} (2p''^2 + 2M^2 - t_1 - t_2) \quad (\text{B2})$$

The following identity holds

$$\begin{aligned} & \frac{s(t_1 + t_2 - t)}{(p''^2 - M^2)(k''^2 - m^2)} - \frac{s - M^2 + m^2}{k''^2 - m^2} - \frac{s + M^2 - m^2}{p''^2 - M^2} = \\ & \frac{\nu^2 - \nu_0^2}{(p''^2 - M^2)(k''^2 - m^2)} \left(-\frac{1}{4} + \frac{1}{2\sqrt{4M^2 - t}} \left[p_0'' - \frac{\nu + 4M^2 - t}{\sqrt{\nu^2 - \nu_0^2}} p_x'' \right] \right) \end{aligned} \quad (\text{B3})$$

For $\nu = \nu_0$ the r.h.s. vanishes. Multiplying the obtained equation by arbitrary function $f(p'')$ and integrating over $d^4 p''$ we obtain the first sought relation

$$\int f(p'') d^4 p'' \left\{ \frac{s(t_1 + t_2 - t)}{(p''^2 - M^2)(k''^2 - m^2)} - \frac{s - M^2 + m^2}{k''^2 - m^2} - \frac{s + M^2 - m^2}{p''^2 - M^2} \right\} \Big|_{\nu=\nu_0} = 0 \quad (\text{B4})$$

To find the relations containing the derivative of I_4 , we divide Eq.(B3) by $\nu^2 - \nu_0^2$, multiply by arbitrary function of the form $f(p''^2, t_1, t_2)$ and integrate over $d^4 p''$, keeping in mind to put $\nu = \nu_0$ afterwards. The r.h.s. will consist of three integrals, the last of which is

$$\frac{1}{\sqrt{\nu^2 - \nu_0^2}} \int \frac{f(p''^2, t_1, t_2)}{p''^2 - M^2} \frac{p_x'' d^4 p''}{s + p''^2 + 2K_x p_x'' - 2(P_0 + K_0)p_0''} \quad (\text{B5})$$

(the long expression in the denominator is equal to $k''^2 - m^2$). Substituting $p_x'' \rightarrow -p_x''$ and averaging obtained and initial integrals, we obtain in the limit $\nu \rightarrow \nu_0$ (which implies $K_x \rightarrow 0$)

$$\frac{1}{\sqrt{\nu^2 - \nu_0^2}} \int \frac{f(p''^2, t_1, t_2)}{p''^2 - M^2} \frac{p_x'' d^4 p''}{k''^2 - m^2} \Big|_{\nu=\nu_0} = - \frac{1}{\sqrt{4M^2 - t}} \int \frac{f(p''^2, t_1, t_2)}{p''^2 - M^2} \frac{p_x'' d^4 p''}{(k''^2 - m^2)^2} \Big|_{\nu=\nu_0} \quad (\text{B6})$$

After some algebra we obtain the second sought relation (the electron mass m was neglected in the numerators)

$$\begin{aligned} & \int f(p''^2, t_1, t_2) d^4 p'' \left\{ \frac{s(t_1 + t_2 - t)}{(p''^2 - M^2)(k''^2 - m^2)} - \frac{s - M^2 + m^2}{k''^2 - m^2} - \frac{s + M^2 - m^2}{p''^2 - M^2} \right\} \frac{t_1 + t_2 - t}{\nu^2 - \nu_0^2} \Big|_{\nu=\nu_0} = \\ & = \frac{1}{4\nu t} \int f(p''^2, t_1, t_2) d^4 p'' \left\{ \frac{t_1 + t_2 - t}{\nu - t} \left[\frac{\nu^2 + t^2}{p''^2 - M^2} - \frac{2\nu t}{k''^2 - m^2} \right] - \frac{2\nu t_1 t_2}{(k''^2 - m^2)(p''^2 - M^2)} + \frac{2t t_1 t_2}{(k''^2 - m^2)^2} \right\} \end{aligned} \quad (\text{B7})$$

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